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## STRUCTURAL LIMITATIONS OF MODEL REFERENCE ADAPTIVE CONTROLLERS

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### ABSTRACT

A general framework is developed for the analysis of performance and robustness properties of tuned adaptive control systems. The analysis is specialized to the case of Model Reference Adaptive Control. It is shown that certain combinations of performance objectives and *a priori* uncertainty lead to unsolvable MRAC design problems, while other combinations lead to problems which can be solved only by careful choice of the reference model.

### I. INTRODUCTION

Much attention has been paid to the questions of stability and parameter convergence of Model Reference Adaptive Controllers (MRAC's). Under certain idealized assumptions, powerful results in these areas have been obtained. More recently, considerable effort has been devoted to these same two questions under weakened assumptions, including the assumption of neglected or "unstructured" plant dynamics and disturbances. Research in this area, under the banner of "robust adaptive control," is extensive.

In spite of this activity, certain other issues of model reference adaptive control have been neglected. Even ignoring the difficulty of achieving a robust adaptation process, the Model Reference approach has certain inherent structural limitations. That is, the "tuned system" to which MRACs converge under a model matching design rule may not have an acceptable level of stability robustness or an acceptable sensitivity function.

This paper is devoted to the study of the "design rule" which calculates controller parameters from (sometimes implicit) plant parameters. A general evaluation framework is formulated, and is applied to the explicit evaluation of a model reference design rule. The specific design rule studied corresponds exactly to the Narendra-Lin-Valavani controller in reference [3] for plants with (nominal) relative degree equal to one.

The paper does not deal with questions of robust identification. Rather, we assume that adaptation is complete, that a plant description has been obtained which is accurate to within a certain tolerance, and that the design rule has been applied to construct the controller. Under these assumptions, we examine the questions of robust stability, nominal performance, and the more advanced question of robust performance. In addition, we illuminate the importance of *a priori* knowledge of the plant description parameters, or "structured uncertainty." The role of the model choice in determining performance and robustness is also exposed.

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Structured singular value analysis proves to be a useful tool in studying these questions. Originally developed for multivariable control performance and robustness analysis in the presence of multiple perturbations [2], the structured singular value allows one to probe the performance of a tuned adaptive system with perturbations included which represent *a priori* structured uncertainty as well as post-adaptation residual or unstructured uncertainty. The utility of this tool is illustrated through specific examples.

Our analyses show that certain levels of structured and unstructured uncertainty preclude successful solution of the design problem via MRAC methods. For these levels, performance and robustness goals cannot be guaranteed under a model matching design rule regardless of the choice of reference model. Alternatively, other levels of uncertainty lead to tractable problems, with performance and robustness goals achievable through careful choice of the model. These results are established both theoretically and through examples.

The paper is organized as follows. In section II, we develop a formal evaluation framework for a general class of design rules. In section III, an MRAC structure is given, and certain manipulations are performed to facilitate its analysis. Section IV provides several theoretical results revealing the importance of *a priori* information in determining achievable levels of nominal performance and robust stability in tuned MRAC systems. Section IV addresses these questions as well as the more advanced question of performance-robustness through the use of structured singular value analysis. Examples are given showing both an unsolvable design problem, and a design problem which is solvable through careful choice of the reference model.

### II. FORMAL DEFINITION OF EVALUATION CRITERIA

#### A. Partitioning of Uncertainty

The true system  $P$  will be assumed to have both structured and unstructured uncertainty. A simple parameterized description of the plant will be assumed. In adaptive control, an on-line identification process determines the parameters of the description, either explicitly or implicitly. The *a priori* uncertainty in the parameter values will be called structured uncertainty. It is recognized that regardless of the choice of the parameters, the plant description will not completely des-

cible the plant dynamics. The residual error will be called unstructured uncertainty.

The relative amounts of structured and unstructured uncertainty depend not only upon *a priori* knowledge of the system, but also upon the system excitation and the performance of the identification mechanism. These issues are quite complex and problem-dependent, and will not be treated here. Instead, we take as a starting point an *a priori* set description of the possible levels of structured and unstructured uncertainty, without regard for its origin, and we study the consequences of various control design rules under the given levels of uncertainty. That is, we will assume

$$P = P_S(c)P_U$$

where  $P_S(c) \in \mathcal{P}_S$   
and  $P_U \in \mathcal{P}_U$

and  $\mathcal{P}_S$  and  $\mathcal{P}_U$  are known sets describing the possible structured plant descriptions  $P_S$  and the residual unstructured plant dynamics  $P_U$ , respectively. Note that the dependence of the plant description on a parameter vector  $c$  has been made explicit.

A variety of uncertainty set characterizations may be chosen. We shall assume a cone of unstructured uncertainty given by

$$\mathcal{P}_U = \{1 + E_U(s) : P \text{ and } P_S \text{ have same number of poles in } \mathcal{C}^+ \\ |E_U(j\omega)| \leq \bar{E}_U(j\omega) \forall \omega, \\ \bar{E}_U(j\omega) \text{ is known a priori} \}$$

This choice is popular in non-adaptive robustness analysis.

For the case of structured uncertainty, we will study multiple characterization options. All are based on the structure  $P_S(c) = k_p N_p(s)/D_p(s)$  where  $N_p$  and  $D_p$  are coprime monic polynomials of known degree  $m$  and  $n$  respectively, with  $N_p$  Hurwitz.  $k_p$  is a gain of known sign. The parameter vector  $c$  represents the value of  $k_p$  and the coefficients of  $N_p$  and  $D_p$ . Global uncertainty will be taken to mean no further *a priori* knowledge of  $c$ . We will also study the case in which additional *a priori* knowledge exists in the form of an estimate of  $D_p$ , denoted  $D_{p0}$ , and a known set of possible deviations  $\{E_S(j\omega)\}$  where  $E_S$  is defined implicitly by

$$D_p(s) = D_{p0}(s)(1 + E_S(s)) \quad (1)$$

Since  $E_S$  represents polynomial uncertainty rather than rational transfer function uncertainty, a conic representation of  $\{E_S(j\omega)\}$  is not natural. Instead we will study two representations — one which contains the set  $\{E_S(j\omega)\}$ , and a second smaller representation which is contained entirely within  $\{E_S(j\omega)\}$ . That is, we shall study the two cones

$$\{E_S(j\omega) : |E_S(j\omega)| \leq \underline{E}_S(j\omega)\} \subset \{E_S(j\omega)\} \quad (2)$$

$$\{E_S(j\omega) : |E_S(j\omega)| \leq \bar{E}_S(j\omega)\} \forall \omega \supset \{E_S(j\omega)\} \quad (3)$$

where  $\underline{E}_S$  and  $\bar{E}_S$  are nonnegative and known *a priori*. An example of the set  $\{E_S(j\omega)\}$  and the two cones are illustrated in Figure 0. The amorphous depiction of  $\{E_S(j\omega)\}$  illustrates that within the above framework lie many characterizations of *a priori* knowledge of  $D_p$ .

## B. Design Rule

The control design rule is the law governing the choice of the feedback compensator given the parameters of the plant description. The feedback compensator may be described by an operator  $K$ :

$$u_p = K \begin{bmatrix} r \\ y_p \end{bmatrix}$$

where  $u_p$  is the plant input,  $y_p$  is the output, and  $r$  is an external command input.  $K$  is determined by a design rule  $f_K$ :

$$K = f_K(c, l)$$

where  $c$  is the vector of plant parameters, and  $l$  is a vector of designer-selectable parameters (for example, the "model" choice in a model reference scheme). Since the goal is to characterize the post-adaptation or "tuned" system,  $K$  may be thought of as a two-input, one-output linear time-invariant transfer function.

We will describe an analysis of various design rules under the uncertainty partitioning shown above. We will also analyze an MRAC design rule explicitly.

## C. Control Design Quality Measures

We will adopt two important and popular indicators of control system quality, namely the sensitivity and complementary sensitivity functions. Classically, these functions are examined in the frequency domain. We will examine them more formally in the weighted  $H_\infty$  seminorm suggested in [5], which is a currently well-accepted approach.

For the system of Figure 1, the sensitivity, denoted  $S(c, l, s)$ , is the transfer function from  $u_1$  to  $e$ , under the design rule  $K = f_K(c, l)$ , assuming  $E_U(s) = 0$ . The sensitivity of a feedback loop is a measure of how its command tracking performance changes under perturbations of the mathematical description. Small sensitivity means that the effect of external disturbances and small plant perturbations on the command tracking accuracy is small, and that minor errors in the plant description also have a small effect on the closed loop response to commands.

We define desired sensitivity properties under a design rule as follows.

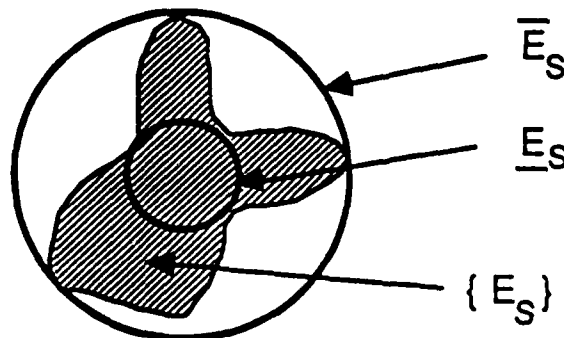


Figure 0  
Conic Cover and Conic Subset of  
Structured Uncertainty

**Figure 3**  
**A Tuned MRAC Structure Representation**

## B. The Design Rule

Since we explicitly allow for perturbations in our analysis, we may define the ideal system by assuming  $E_U = 0$  and  $\Lambda_1 = 1$ . We shall define the tuned system under these assumptions, and later include  $E_U$  and  $\Lambda_1$  as perturbations in the analysis of the system. (Note: in the case of relative degree 1 nominal plant descriptions,  $\Lambda_1$  is identically 1, and only  $E_U$  is neglected.)

Under the assumptions  $E_U = 0$  and  $\Lambda_1 = 1$ , the MRAC design rule enforces a model-matching criterion given by

$$\frac{c_p k_p N_p(s) N_M(s)}{D_p(s)(N_M(s) + C(s)) - k_p N_p(s) D(s)} = \frac{k_M N_M(s)}{D_M(s)} \quad (4)$$

The design rule  $f_K$  is defined implicitly by this equation.

## C. Algebraic Implications of the Design Rule

It can be proven (e.g. in [4]) that the model-matching criterion uniquely defines  $c_p$ ,  $C(s)$  and  $D(s)$  in terms of the plant and model parameters. Unfortunately, the relationship of  $c_p$ ,  $C$ , and  $D$  to the plant and model is not very transparent. In order to perform a study of the implications of the design rule on performance and robustness, some simplifications are required. This section describes the algebraic manipulations which accomplish this simplification.

First, note that the numerator and denominator of the left-hand-side of (4) have degree greater than the numerator and denominator of the right hand side. It follows that  $m$  pole-zero cancellations occur. Equating the numerators reveals that the cancellations involve the roots of  $N_p$ . One finds that the design rule is equivalent to the following rules:

$$c_p k_p = k_M \quad (5)$$

$$D_p(s)(N_M(s) + C(s)) - k_p N_p(s) D(s) = N_p(s) D_M(s) \quad (6)$$

Equation (6) is a polynomial equality valid for all  $s$ . One can simplify the equations by examining interesting choices of  $s$ . In particular, let  $s_i$  be a zero of  $N_p(s)$ . Then equation (6) becomes

$$D_p(s_i)(N_M(s_i) + C(s_i)) = 0 \quad (7)$$

Since  $N_p$  and  $D_p$  are coprime,  $D_p(s_i)$  is not zero, hence

$$N_M(s_i) + C(s_i) = 0 \quad (8)$$

Since this is true for all  $m$  zeros of  $N_p$ , and since  $N_M + C$  has exactly  $m$  zeros,

$$N_M(s) + C(s) = N_p(s) \quad (9)$$

Substituting (9) into (6) and dividing by  $N_p$  one obtains

$$k_p D(s) = D_p(s) - D_M(s) \quad (10)$$

Examination of Figure 3 in light of equations (9) and (10) reveals the sensitivity, complementary sensitivity, and loop transfer functions:

$$S(s) = \frac{D_p(s) \Lambda_1(s)}{D_p(s)(\Lambda_1(s) - 1) + D_M(s)} \quad (11)$$

$$T(s) = \frac{D_p(s) - D_M(s)}{D_p(s)(\Lambda_1(s) - 1) + D_M(s)} \quad (12)$$

$$L(s) = \frac{D_p(s) - D_M(s)}{D_p(s) \Lambda_1(s)} \quad (13)$$

Note that further simplification occurs in the case of relative degree 1 nominal plants, since then  $\Lambda_1 = 1$ .

## IV. EVALUATION OF THE MRAC STRUCTURE

### A. Conditions for Robust Stability

#### 1. Robustness Under Global Uncertainty

In [3] a design rule similar the one studied here (exactly the same when  $n-m=1$ ) was proven to be globally stable. That is, the closed loop system was stable for any value of the coefficients of  $N_p(s)$  and  $D_p(s)$ , subject to a minimum phase requirement on  $N_p$ . Taking this same structured uncertainty as  $P_S$  in our analysis, we find that the robustness margins with this design rule are extremely small.

**Theorem 1:** Under global structured uncertainty, if the unstructured uncertainty bound  $\bar{E}_U(j\omega)$  is strictly greater than  $(\Lambda_1(j\omega) - 1)$  for any  $\omega$ , the MRAC design rule is not *potentially robustly stabilizing*.

**Proof:** Examination of (12) reveals that as one allows the magnitude of  $D_p(s)$  approach infinity, keeping other coefficients fixed,  $|T(j\omega)| \rightarrow \frac{1}{\Lambda_1(j\omega) - 1}$ . Thus

$$c: P_S \in \mathcal{P}_S \{ |T(c, j\omega)| \bar{E}_U(j\omega) \} = \left| \frac{1}{\Lambda_1(j\omega) - 1} \right| \bar{E}_U(j\omega)$$

Since this is true for each  $l$ ,

$$\inf_l \left\{ \sup_{c: P_S \in \mathcal{P}_S} \{ |T(c, j\omega)| \bar{E}_U(j\omega) \} \right\} = \left| \frac{1}{\Lambda_1(j\omega) - 1} \right| \bar{E}_U(j\omega)$$

Lemma 1 completes the proof.  $\square$

Since  $\Lambda_1$  is approximately 1 over the frequency range of interest in the control problem, the stability margin is small. Moreover, we have as a corollary that the margin is zero in the case of relative degree one plants:

**Corollary:** For relative degree one plants, the MRAC design rule is not potentially robustly stabilizing under global structured uncertainty unless the unstructured dynamics are assumed to be exactly zero.

#### 2. Robustness Under Bounded Uncertainty

Even when the structured uncertainty is not global, there may be no choice of the reference model which results in robust stability.

Consider the case of a relative-degree-one plant with denominator uncertainty including a known cone, as given by (2).

**Theorem 2:** Under these conditions, a *necessary* condition for *potential robust stability* is:

$$\bar{E}_S(j\omega) \bar{E}_U(j\omega) < 1 \quad \text{for all } \omega \text{ where } \bar{E}_U(j\omega) > 1$$

Proof: Regardless of  $D_M$ , extremizing equation (12) subject to  $|E_S(j\omega)| < \bar{E}_S$  yields

$$\inf_{c: P_S \in \mathcal{P}_S} \left\{ \sup_{c: P_S \in \mathcal{P}_S} |T(c, j\omega)| \bar{E}_U(j\omega) \right\} = \frac{|D_{P0} - D_M| + \bar{E}_S |D_{P0}|}{|D_{P0} - D_M - D_{P0}|} \bar{E}_U$$

$$\geq \frac{|D_{P0} - D_M| + \bar{E}_S |D_{P0}|}{|D_{P0} - D_M| + |D_{P0}|} \bar{E}_U$$

Now, by the methods of calculus one can infimize this last quantity over all  $D_M(j\omega)$  with the result that at the infimum, the inequality becomes an equality, and one has

$$\inf_{c: P_S \in \mathcal{P}_S} \left\{ \sup_{c: P_S \in \mathcal{P}_S} |T(c, j\omega)| \bar{E}_U(j\omega) \right\} = \begin{cases} \bar{E}_U(j\omega) & \text{if } \bar{E}_S(j\omega) \geq 1 \\ \bar{E}_S(j\omega) \bar{E}_U(j\omega) & \text{if } \bar{E}_S < 1 \end{cases}$$

which is logically equivalent to the theorem.  $\square$

This theorem implies that for some combinations of structured and unstructured uncertainty one cannot guarantee stability by choice of the model. Furthermore, as a minimal requirement for stability guarantees, one must know the structured plant denominator well *a priori* in those frequency ranges where large robustness margins are desired.

The theorem only provides a necessary condition for two reasons. First, an optimization was performed pointwise at each frequency to determine (or at least limit)  $D_M(j\omega)$ . However there is no guarantee that there exists a Hurwitz polynomial of degree  $n$  which has exactly this frequency response. Since the only allowable choices for  $D_M$  in the MRAC structure are Hurwitz and degree  $n$ , there is perhaps no model which provides robust stability even when Theorem 2 is satisfied. Second, the lower bound in (2) need not represent the complete family of possible plants.

Somewhat stronger conditions are actually sufficient for robust stability.

**Theorem 3:** For a relative degree one plant and a conic bounded structured uncertainty ball  $D_P = D_{P0}(1 + E_S)$ , a sufficient condition for *potential robust stability* is:

$$D_{P0}(s) \text{ is Hurwitz, and}$$

$$\bar{E}_S(j\omega) \bar{E}_U(j\omega) < 1 \quad \forall \omega$$

Proof: Choosing  $D_M = D_{P0}$  and applying equation (12) yields

$$\sup_{c: P_S \in \mathcal{P}_S} |T(j\omega, c, D)| \bar{E}_U(j\omega) = \bar{E}_S(j\omega) \bar{E}_U(j\omega)$$

The theorem then follows from the definition of robust stability.  $\square$

It is worth noting the sacrifice that was made in achieving robust stability in this theorem. A consequence of choosing the model poles to be the *a priori* nominal plant poles is poor sensitivity. For example, when the plant happens to be exactly the *a priori* estimate, the sensitivity is one at all frequencies (equation (11) with  $\Lambda_1 = 1$ ,  $D_P = D_M = D_{P0}$ ). Such issues motivate the study of simultaneous performance and robustness through a single model choice, which is treated later.

## B. Conditions for Desensitivity

### 1. Desensitivity Under Global Uncertainty

**Theorem 4:** If the sensitivity specification  $|E_P(j\omega)|$  is strictly greater than  $(\Lambda_1(j\omega) - 1)/\Lambda_1(j\omega)$  for any  $\omega$ , then for any choice of the model, the system is not *desensitizing* under global uncertainty.

Proof: Using the sensitivity expression (11), the definition of desensitivity becomes

$$\sup_{D_P(j\omega)} \left\{ \left| \frac{D_P(j\omega) \Lambda_1(j\omega)}{D_P(j\omega) (\Lambda_1(j\omega) - 1) + D_M(j\omega)} \right| \bar{E}_P(j\omega) \right\} < 1 \quad \forall \omega$$

The theorem follows by letting  $|D_P|$  approach infinity.  $\square$

Corollary: For relative degree one plants, no design is desensitizing when  $E_P(j\omega) > 0$  for any  $\omega$ .

### 2. Desensitivity Under Bounded Uncertainty

**Theorem 5:** For the case of a relative degree one plant and a conic bounded structured denominator uncertainty, the MRAC design rule is always *potentially desensitizing*, provided that  $(1 + \bar{E}_S(j\omega)) \bar{E}_P(j\omega) < 1$  for sufficiently large  $\omega$ . (This is a very reasonable requirement.) Moreover, a *sufficient* condition for a given model choice to be desensitizing is

$$\left| \frac{D_{P0}(j\omega)}{D_M(j\omega)} \right| (1 + \bar{E}_S(j\omega)) \bar{E}_P(j\omega) < 1 \quad \forall \omega$$

Proof: Applying the sensitivity expression (11) and the conic bound on  $D_P$  to the definition of desensitivity, one finds desensitivity is implied by

$$\sup_{|E_S(j\omega)| < \bar{E}_S(j\omega)} \left\{ \left| \frac{D_{P0}}{D_M} \right| (1 + E_S) \bar{E}_P \right\} < 1 \quad \forall \omega$$

The fact that the structure is always *potentially desensitizing*, i.e., that there always exists some model satisfying the condition of the theorem, follows by simply letting  $|D_M|$  approach  $\infty$ .  $\square$

## IV. COMBINED PERFORMANCE AND ROBUSTNESS

While previous sections have examined questions of desensitivity and stability robustness, in practice one is not satisfied with a controller that has only one of these properties — it must have both.

In this section, we describe a general analysis technique for tuned adaptive systems. The technique is appropriate for addressing the separate questions of stability robustness and desensitivity, as well as the more advanced question of robust performance. Robust performance implies not only a guarantee of stability under all structured and unstructured uncertainty, but also a guarantee of a certain level of desensitivity under the entire set of uncertainty.

The theoretical foundation both for the definition of robust performance and its evaluation, is the structured singular value theory ([1], [2]).

## A. The Design Specification

The question of whether a desensitivity specification is satisfied under system uncertainty may be transformed into a robust stability analysis problem ([1]). The technique involves introducing an additional unstructured uncertainty  $E_p(s)$  into the system, with the input to  $E_p$  being  $e_1$  of Figure 1, and the output being the  $y_p$  node of Figure 1. One assumes that the perturbation  $E_p$  is stable but unknown except for the bound  $|E_p(j\omega)| < \bar{E}_p(j\omega) \forall \omega$ . Here  $\bar{E}_p$  is the desensitivity specification which appears in definition 1. If the system is robustly stable under all uncertainties including this added (fictitious) uncertainty, then the actual system satisfies the desensitivity spec (definition 1) under the entire set of actual uncertainty ([1]).

For the special case of the structure at hand, one may use the algebraic results of section III to manipulate the tuned system representation, obtaining the equivalent representation of Figure 4. In Figure 4, the added uncertainty block  $E_p$  has been included as described above. In addition, we have defined the structured uncertainty  $E_s$  in the figure implicitly by  $D_p(s) = D_{p0}(s)(1 + E_s(s))$ , as described previously.

One specifies the control system objectives by specifying the size of the uncertainties, and the desired performance under these uncertainties.

**Definition 5:** The *Design Specification* is formed by the sets describing  $E_s$ ,  $E_U$ , and  $E_p$ , with the interpretation that one requires the desensitivity indicated by  $E_p$  be guaranteed under the uncertainty indicated by  $E_s$  and  $E_U$ .

**Definition 6:** The *Design Measure* is the inverse of the largest factor by which all of the uncertainty sets in Figure 4 can be scaled while retaining a guarantee of stability of the system of Figure 4.

By the above discussion, "Design Measure  $\leq 1$ " indicates that the Design Specification is met. "Design Measure  $> 1$ " indicates that the Design Specification is not met.

## B. Structured Singular Value Analysis

Structured singular value analysis was originally developed for the evaluation of multivariable control system robustness. As we shall show in examples to follow, the structured singular value has importance in analysis of tuned adaptive control systems as well.

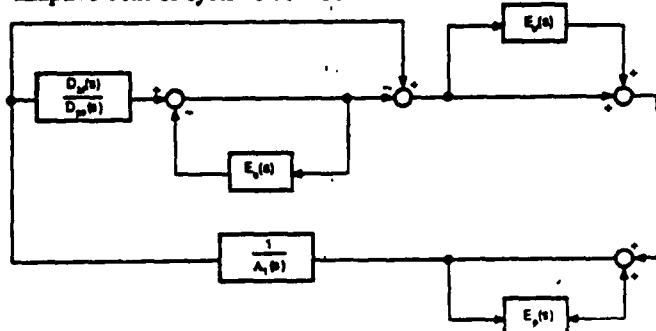


Figure 4  
An Alternate Tuned MRAC Representation

Evaluation of the Design Measure is aided by existing structured singular value analysis software. Currently, the software can efficiently evaluate the stability of the general system of Figure 5, where  $M(s)$  is a known transfer function, and the  $\Delta_i: \mathcal{C} \rightarrow \mathcal{C}$  are both analytic and bounded by one in the right half plane, but otherwise unknown. For perturbations of size other than one, the uncertainties may be factored into a unity-bounded parts and known scaling factors. The scaling factors may then be absorbed into the system  $M(s)$ . Such operations are routine in multivariable control.

The output of the software is a real number  $\mu$  for each frequency  $\omega$ , equal to the reciprocal of the largest factor by which all the  $\Delta_i$ 's can be scaled before  $\det[I - \text{diag}(\Delta_i(j\omega))M(j\omega)]$  vanishes. Thus, if the maximum of  $\mu$  over all frequencies is less than unity, the system remains stable, while if the maximum exceeds unity, it will be unstable for some value of the  $\Delta_i$ 's. It follows that  $\mu$  provides a direct evaluation of our Design Measure, provided only that the three uncertainty blocks in Figure 4 are unstructured.

Of course,  $E_s$  is *not* unstructured. Nevertheless, the structured singular value analysis may still be applied to the tuned MRAC through a two-step procedure.

First, the cone covering  $\{E_s(j\omega)\}$  (radius =  $\bar{E}_s$ ) is used. The structured singular value analysis then determines if the system of Figure 4 is stable with this cone of uncertainty. If so, it is also stable with the smaller set  $\{E_s\}$ , which implies that the Design Specification is met. Letting  $\bar{\mu}$  represent the output of this structured singular value analysis,  $\sup_{\omega} \bar{\mu}$  is an upper bound on the Design Measure.

Second, a cone within  $\{E_s(j\omega)\}$  (radius =  $\underline{E}_s$ ) is used. It is a minimal requirement that this smaller uncertainty be tolerable. Performing a structured singular value analysis and denoting the output  $\underline{\mu}$ ,  $\sup_{\omega} \underline{\mu}$  is a lower bound on the Design Measure. Clearly, the Design Specification is violated if  $\underline{\mu}$  is greater than one at any frequency.

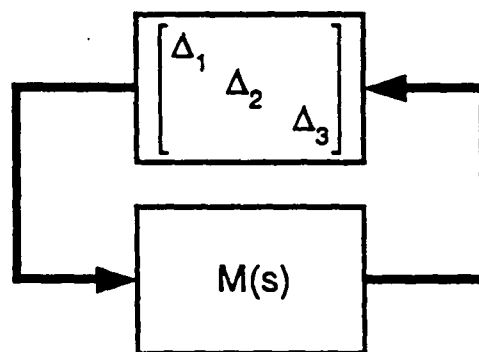


Figure 5  
The Structured Singular Value Problem Setting

## C. Examples

### 1. A Solvable MRAC Design Problem

Consider a nominal plant with  $\text{degree}(D_P) = 2$ , and  $\text{degree}(N_P) = 1$ , and, thus, a relative degree of 1. Let the model denominator be  $D_M = s^2 + 20s + 100$ . Since the plant and model numerators do not enter into our analysis, we need only assume that both have roots in the open left half plane. Let an *a priori* estimate of the plant denominator be  $D_{P0} = s^2 + 8s + 16$  with the *a priori* error bound:

$$D_P(s) - D_{P0}(s) = \delta_1 s + \delta_2 \text{ where } 100\delta_1^2 + \delta_2^2 \leq 400. \quad (14)$$

Defining  $E_S$  implicitly by  $D_P(s) = D_{P0}(s)(1 + E_S(s))$  yields  $E_S(s) = (\delta_1 s + \delta_2)/(s^2 + 8s + 16)$ . Then for each  $\omega$ , the set of structured uncertainty  $\{E(j\omega)\}$  implied by equation (14) is an ellipse with major (minor, respectively) radii  $\bar{E}_S(j\omega)$  ( $\underline{E}_S(j\omega)$ ) =

$$\max(\min, \text{resp.}) \left\{ \left| \frac{2s}{s^2 + 8s + 16} \right|, \left| \frac{20}{s^2 + 8s + 16} \right| \right\}$$

Here  $\bar{E}_S$  and  $\underline{E}_S$  serve as radii for a conic cover and a conic subset, as described in section II. Thus one can describe an inscribed cone and a conic bound by

As the Design Specification, let the size of the unstructured dynamics to be tolerated, and the desired desensitvity be given by:

$$E_U(s) = \Delta_2(s)W_U(s), \quad W_U(s) = \frac{5s + 20}{s + 200}$$

$$E_P(s) = \Delta_3(s)W_P(s), \quad W_P(s) = \frac{0.5(s + 5)}{s + 1}$$

Using the conic cover  $\bar{E}_S$ , and performing a structured singular value analysis, we obtain an upper bound on the Design Measure, shown in Figure 6. Since the Design Measure is less than one everywhere, the MRAC controller designed for the specified  $D_M$  will satisfy the Design Specification (if it converges).

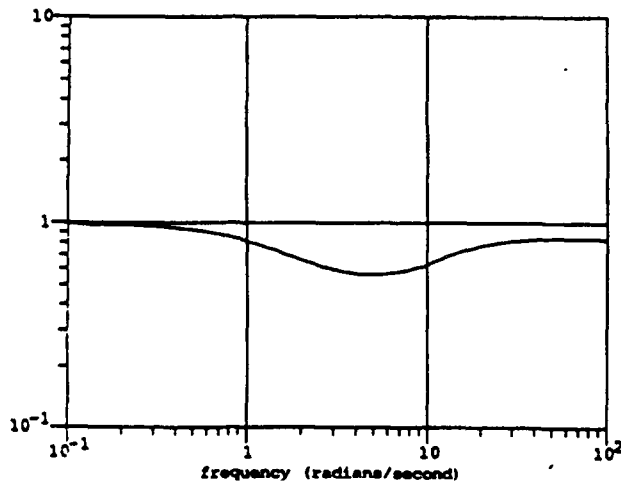


Figure 6  
An Upper Bound on the Design Measure  
For a Successful Design

It is worth noting that this analysis is an entirely *a priori* analysis of the full set of possible post-adaptation systems.

One may perform this *a priori* analysis for a variety of designs. It is reasonable to assume that the desensitvity requirements as well as the size of the structured and unstructured uncertainty sets are fixed features of a design task. The design parameter which may be chosen is the reference model, through which the tuned controller is implicitly designed.

The separate performance and robustness properties studied in section IV, as well as the requirement that the model be stable, provide limits on the choice of models that need be considered. Within these limits, a model choice may or may not satisfy the more challenging and more important goal of robust performance under the full set of uncertainty. For each choice of the model, one may use the structured singular value analysis to classify the design as either (1) satisfying the design specification, (2) violating the design specification, or (3) indeterminate through the structured singular value analysis.

Keeping all but the model choice the same as above, a classification of the design has been performed for a grid of possible model choices, with the results shown in Figure 7. The figure illustrates the fact that standard tuned model reference systems can satisfy control objectives in some problem settings, provided that the model is carefully chosen.

### 2. An Unsolvable MRAC Design Problem

With precisely the same performance objective and the same set of unstructured uncertainty as in the previous example, the design task becomes intractable when the *a priori* uncertainty becomes too large. Let the size of  $\{E_S(j\omega)\}$  become larger by a factor of 10. This corresponds to the *a priori* information

$$100\delta_1^2 + \delta_2^2 \leq 40000. \quad (15)$$

Repeating the procedure of the first example, but using the inscribed uncertainty cone in the structured singular value analysis to obtain a lower bound on the design margin, one obtains the result shown in Figure 8. Since  $\mu > 1$  at some frequencies, the design does not satisfy the Design Specification.

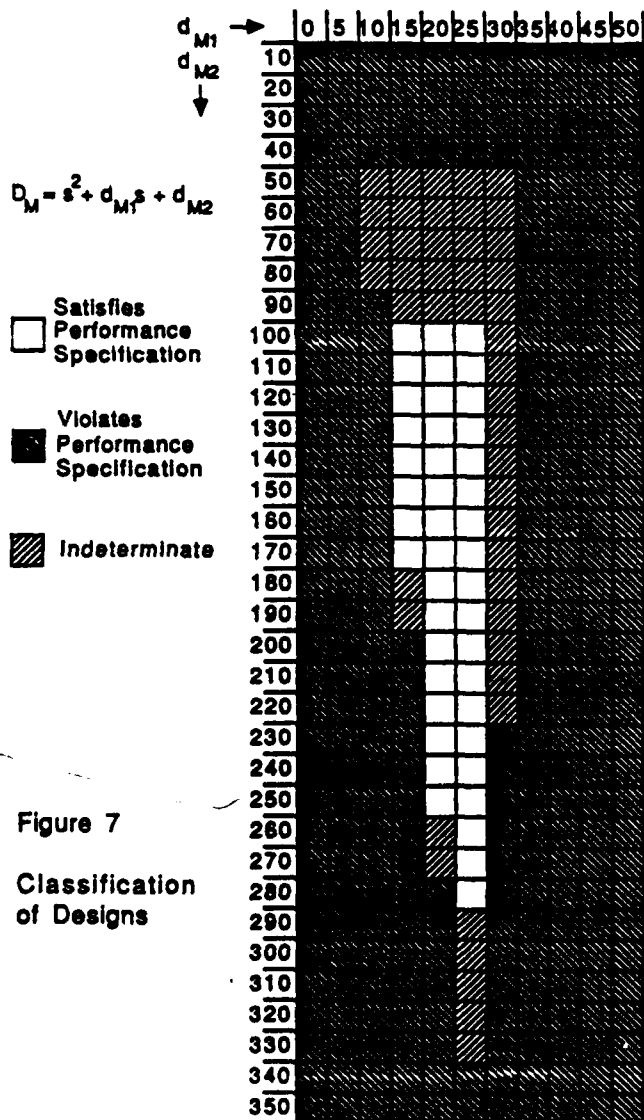
Exploration of alternate model choices indicates that one cannot meet the Design Specification through choice of the design parameter. The corresponding figure to Figure 7 would show all designs to be classified as violating the spec.

This example illustrates that the standard model reference adaptive control structure cannot solve certain practical control design tasks, even under the greatly simplifying assumption that successful identification/adaptation does take place.



## V. CONCLUSIONS.

When one takes into consideration the usual performance and robustness requirements of a control system, a model reference adaptive control problem may be ill-posed in the sense that the tuned system indicated by the model matching design rule may not be acceptable. However, through the analysis framework of this paper, one can determine whether a design rule will lead to an acceptable system, assuming convergence is achieved. Only in such cases need one be concerned with the (formidable) question of robust identification.



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